Final Exam - Review 2 - Problems

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1 Definition of the integral

Problem 1:

Evaluate $\int_1^2 x^2 dx$ using the **definition** of the integral. You may use the facts that $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ and $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$.

Problem 2:

Use the midpoint rule with $n = 4$ to approximate $\int_1^2 e^{-x^2} dx$

2 Fundamental theorem of calculus

Problem 3: Differentiate $g(x) = \int_{e^x}^{\sin(2x)} \sin^{-1}(t) dt$

Problem 4: Evaluate:

(a) $\int \frac{d}{dx} \sqrt{|\sin(x) + \cos(x)|} dx$ and (b) $\frac{d}{dx} \int \sqrt{|\sin(x) + \cos(x)|} dx$

Problem 5: Evaluate the following integrals:

(a) $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$

(b) $\int_{-1}^1 \tan\left(\frac{\pi}{4} x^3\right) dx$

(c) $\int_0^2 \sqrt{4 - x^2} dx$

(d) $\int \frac{x^3}{1+x^4} dx$

(e) $\int \frac{1+x}{x^2+1} dx$

(f) $\int_0^1 \frac{\tan^{-1}(x)}{1+x^2} dx$

(g) $\int \frac{dx}{9x^2+1}$

(h) $\int_1^2 x \sqrt[4]{x-1} dx$

3 Areas

Problem 6

Find the area of the region bounded by the parabola $y = x^2 - 4$ and the two lines connecting $(-2, 0)$ with $(0, \frac{1}{2})$, and $(0, \frac{1}{2})$ with $(2, 0)$

4 Volumes

Problem 7

Show that the volume of a ball with radius R is $\frac{4}{3}\pi R^3$

Problem 8

Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = x^3$ about $y = 2$

Problem 9

Find the volume of the solid obtained by rotating the region bounded by $y = 2x^2$ and $y = 3x - x^2$ about the $y-$axis

Problem 10

Find the volume of a cone with radius r and height h

Problem 11

Find the volume of the torus/donut obtained by rotating the disk of center $(2, 0)$ and radius 1 about the $y-$axis.